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The analysis of the found girder on the soil of general stratification

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The work shows the results of the investigation of contact problems in the system of transverse loaded flexible foundation girder and compressible soil with general stratification. The results of the accomplished theoretical analysis and those of the model research conform. The developed program can be explicitly useful in designing long foundation girders, for which the varying composition of soil along the girder is a common occurrence.
Keywords: Found girder, soil of general stratification, analysis, experiment.

Introduction

On the girders that lie on the soil, an activeness of unknown reactive load occurs, as a reaction to the activeness of a known outer load, whose total amount is known, but its distribution along the girder is not. At the moment, the available commercial programs on the market, for girders on soil design, have some limitations in the behaviour of the girder and soil. These programs solve only the plane (plane deformations) or axial-symmetrical problems (for a unit angle expressed by arch measure), which have a significant influence on the total distribution of reactive pressures and, ultimately affect the movement and inner forces in the girder.

The shown compute procedure includes the development of algorithms for girder-soil contact problem analysis. As a base for the development of the procedure, the differential equation of transverse loaded girders may be used, in which the bend angle equals the soil settlement, for an undetermined distribution of outer force along the girder. For the numerical solution, a compatible program for computer designing has been made. The proposed procedure was submitted to a model check, which consisted of observing the behaviour of a girder on stratified soil, in different cases of load and soil stratification (linear and leap variation of deformable soil layer thickness.

The foundation girder design

The contact problems were a subject of investigation for many investigators, till today. As the oldest example of solving the contact problems, the investigation of a stiff circled disk behaviour on a homogeneous, elastic and isotropic half space can be stated (Boussinesq 1885). Hertz (Hertz 1882) has solved the contact problem of two elastic bowls with different diameters. The investigation of contact problems of two bodies with complex geometry started in the beginning of the 20th Century. Winkler (Winkler 1867) solved the problem of girders on soil underneath building columns, or the problem of longitudinal tracks for level luffing crane, and the procedure was further developed by Zimmermann (Zimmermann 1888) (so called one-parameter model or Winkler soil model). The basic assumption of this model is very unrealistic (soil modelled as a system of undependable springs), but due to the simple mathematical interpretation of the problem, the model is used even today in engineer practice.

The two-parameter model (Vlasov & Leontiev 1966) is performed from the basic Winkler model. This model enables a connection of Winkler springs with the stretched elastic membrane, wherewith it is possible to model the attribute of soil as a connected continuum. At the one-parameter and two-parameter soil models, the basic problem is their selection (the parameters are not physical and they depend on the amount of load and the shape of the loaded area).

Finally, due to the development of computers and numerical procedures (a method of finite elements, a method of finite differentials and so on), today, it is possible to model complex relations of construction and soil interaction.

The proposed procedure of foundation girder design

The method of finite differentials by which the mathematical discretization of the differential equation of the problem is used. The solution leads to seeking unknown values in discrete points.

Figure 1 shows a discretization of the beam girder with unknown values of movement vector{w} and reactive pressures {q} with a known vector of outer load {p}, and the known weight of a girder segment g.

Differential equation of the girder for point i is:

$$\frac{EI}{a^4}\left(w_{i-2} - 4w_{i-1} + 6w_i - 4w_{i+2} + w_{i+2}\right) + q_i = p_i + g \qquad (1)$$

In the matrix form 1:

$$[D]\{w\} + [\lambda]\{q\} = \{f\} \qquad (2)$$

where:

[D] - matrix of deferential operator,
[λ] - diagonal matrix,
{w} unknown vector of movement,
{q} unknown vector of soil reactive pressure,
{f} - outer acting vector.

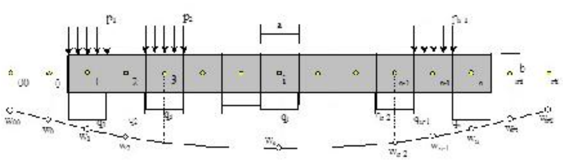


Figure 1. Discretization of the beam girder on n equal elements

The amounts of unknown fictive movements (w_{00}, w_0, w_{n+1} and w_{n+2}) are determined from the edge conditions (known bending moments M_1 and M_n and transverse forces Q_1 and Q_n in points $_1$ and $_n$) using the deferential relations of the bending moment and transverse forces with unknown movements.

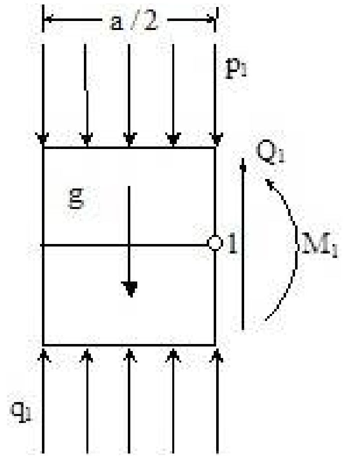


Figure 2. Edge conditions in point 1

$$M_1 = \frac{a^2}{8}(q_1 - g - p_1) \approx -\frac{EI}{a^2}(w_0 - 2w_1 + w_2) \qquad (3)$$

$$Q_1 = \frac{a}{2}(q_1 - g - p_1) \approx -\frac{EI}{2a^3}(-w_{00} + 2w_0 - 2w_2 + w_3) \qquad (4)$$

A differential equation for point 1 can be computed inserting w_0 and w_{00} according to the terms 3. and 4; :

$$\frac{EI}{a^4}(w_1 - 2w_2 + w_3) + \frac{9}{8}q_1 = \frac{9}{8}(g + p_1) \qquad (5)$$

Analogy for point 2:

$$\frac{EI}{a^4}(-2w_1 + 5w_2 - 4w_3 + w_4) - \frac{1}{8}q_1 + q_2 = \frac{7}{8}g - \frac{1}{8}p_1 + p_2 \qquad (6)$$

The deferential matrix operator:

$$|D| = \frac{EI}{a^4}\begin{vmatrix}
1 & -2 & 1 & & & & & & & \\
-2 & 5 & -4 & 1 & & & & & & \\
1 & -4 & 6 & -4 & 1 & & & & & \\
 & 1 & -4 & 6 & -4 & 1 & & & & \\
 & & \cdot & \cdot & \cdot & \cdot & \cdot & & & \\
 & & & 1 & -4 & 6 & -4 & 1 & & \\
 & & & & 1 & 4 & 6 & 4 & 1 & \\
 & & & & & 1 & -4 & 6 & -4 & 1 \\
 & & & & & & 1 & -4 & 5 & -2 \\
 & & & & & & & 1 & -2 & 1
\end{vmatrix}$$

The diagonal matrix $[\lambda]$ has a form:

$$[\lambda] = \begin{bmatrix} \frac{9}{8} & & & & & & & & & \\ \frac{1}{8} & 1 & & & & & & & & \\ & & 1 & & & & & & & \\ & & & 1 & & & & & & \\ & & & & \cdot & & & & & \\ & & & & & 1 & & & & \\ & & & & & & \cdot & & & \\ & & & & & & & 1 & & \\ & & & & & & & & 1 & \\ & & & & & & & & & 1 & -\frac{1}{8} \\ & & & & & & & & & & \frac{9}{8} \end{bmatrix}$$

The soil reactive vector pressures (q) and loads (f):

$$\{q\} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ \cdot \\ \cdot \\ q_i \\ \cdot \\ \cdot \\ q_{n-2} \\ q_{n-1} \\ q_n \end{Bmatrix} \qquad \{f\} = \begin{Bmatrix} \frac{9}{8}g + \frac{9}{8}p_1 \\ \frac{7}{8}g - \frac{1}{8}p_1 + p_2 \\ g + p_3 \\ \cdot \\ \cdot \\ g + p_i \\ \cdot \\ \cdot \\ g + p_{n-2} \\ \frac{7}{8}g + p_{n-1} - \frac{1}{8}p_n \\ \frac{9}{8}g + \frac{9}{8}p_n \end{Bmatrix}$$

The algebra equation linear system form (2) of n×n order is undefined because there are two vectors {w} and {q} with 2×n unknown components. In order to solve the system, it is necessary to define the additional relation of two unknown vectors (soil modulus) by defining the settlement matrix.

Defining the influence settlements

The value of settlement in the observed point is defined by knowing the distribution of additional stresses and deformable characteristics of the soil under the point.

$$w = \sum \Delta w = \sum \frac{\sigma_v \cdot \Delta z}{M_v} \tag{7}$$

where:

Δw - the increment of settlement,
σ_v - additional vertical stress,
Δz - soil layer thickness,
M_v - layer modulus of incompressibility Δz.

The additional values of vertical stresses σ_v underneath the observer point on the depth z are determined using Steinbrenners solution (Steinbrenner 1934) for the uniform contact pressure q acting on a rectangular with sides a and b.

$$\sigma_z = \frac{q}{2\pi}\left[\operatorname{arctg}\frac{a \cdot b}{z \cdot \sqrt{a^2+b^2+z^2}} + \frac{a \cdot b \cdot z}{\sqrt{a^2+b^2+z^2}}\left(\frac{1}{a^2+z^2}+\frac{1}{b^2+z^2}\right)\right] \tag{8}$$

Figure 3 shows a quality review of vertical stresses on the depth along the perpendicular under the central point of j element due to the uniformly spread load on the a×b area of i element obtained by super positioning the term proper rectangular area (8).

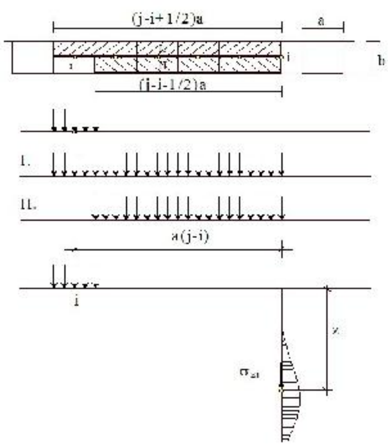


Figure 3. Vertical loads under the central point of j element due to the load on point i

$$\frac{q}{\pi}\left[\operatorname{arctg}\frac{\left(j-i+\frac{1}{2}\right)a \cdot \frac{b}{2}}{z \cdot \sqrt{\left(\left(j-i+\frac{1}{2}\right)a\right)^2+\left(\frac{b}{2}\right)^2+z^2}} + \frac{\left(j-i+\frac{1}{2}\right)a \cdot \frac{b}{2} \cdot z}{\sqrt{\left(\left(j-i+\frac{1}{2}\right)a\right)^2+\left(\frac{b}{2}\right)^2+z^2}}\left(\frac{1}{\left(\left(j-i+\frac{1}{2}\right)a\right)^2+z^2}+\frac{1}{\left(\frac{b}{2}\right)^2+z^2}\right)\right] -$$
$$-\frac{q}{\pi}\left[\operatorname{arctg}\frac{\left(j-i-\frac{1}{2}\right)a \cdot \frac{b}{2}}{z \cdot \sqrt{\left(\left(j-i-\frac{1}{2}\right)a\right)^2+\left(\frac{b}{2}\right)^2+z^2}} + \frac{\left(j-i-\frac{1}{2}\right)a \cdot \frac{b}{2} \cdot z}{\sqrt{\left(\left(j-i-\frac{1}{2}\right)a\right)^2+\left(\frac{b}{2}\right)^2+z^2}}\left(\frac{1}{\left(\left(j-i-\frac{1}{2}\right)a\right)^2+z^2}+\frac{1}{\left(\frac{b}{2}\right)^2+z^2}\right)\right] \tag{9}$$

After determining the values of vertical stresses along the vertical, through every knot i (i=1, 2, 3,... n), it is possible to, knowing the deformable characteristics of every layer, determine the values of influence settlements. The elements of the influence settlements matrix [U] are computed from the following terms:

$$\alpha_{ii} = \int_0^{h_i} \frac{\sigma_z dz}{M_{vz}} = \frac{1}{M_{v1}}\sum_{z=0}^{zi1}\sigma_{zi}\Delta z + \frac{1}{M_{v2}}\sum_{zi1}^{zi2}\sigma_{zi}\Delta z + \dots + \frac{1}{M_{vm}}\sum_{zi,m-1}^{zim}\sigma_{zi}\Delta z \tag{10}$$

$$\alpha_{ji} = \int_0^{h_j} \frac{\sigma_z dz}{M_{vz}} = \frac{1}{M_{v1}} \sum_{z=0}^{zj1} \sigma_{zj} \Delta z + \frac{1}{M_{v2}} \sum_{zj1}^{zj2} \sigma_{zj} \Delta z + \ldots + \frac{1}{M_{vm}} \sum_{zj,m-1}^{zjm} \sigma_{zj} \Delta z \tag{11}$$

Finally, the girder settlement equation for a known reactive pressure distribution is:

$$\{w\} = [U]\{q\} \tag{12}$$

The course of found girder design

By inserting the equation (3) in the differential equation (2), a following term can be obtained:

$$([D]\,[U] + [\lambda])\{q\} = \{f\} \tag{13}$$

The term (13) presents a system of equations in which only the soil reactive movements on the girders are unknown. The final term for solving the pressure reactive vectors {q} is:

$$\begin{aligned} &\underbrace{([D]\,[U] + [\lambda])}_{[\Lambda]}\{q\} = \{f\} \\ &[\Lambda]\{q\} = \{f\} \\ &\{q\} = [\Lambda]^{-1}\{f\} \end{aligned} \tag{14}$$

After solving the equation system (14), the settlement of the knot point can be computed using the term (12). When the movement vector is determined, the vectors of the bending moment and transverse forces can be determined using the terms (3) and (4). Based on the presented procedure a program, named ''NOSAČ99'' (GIRDER99), for designing the foundation girder on a generally stratified soil is made. The solutions gained using the program are related with the results of model testing, which will be shown later in the text.

As it can be seen from the presented procedure, the calculation of the influence settlement matrix is gained using the hybrid method. The Steinbrenner solution for homogeneous isotropic linear elastic half space is used for determining the distribution of additional stresses in the centre of discrete areas (Steinbrenner 1934). The assumption that the distribution of additional stresses does not depend on the soil stratification is accepted (figure 4). In that case, the additional stresses under the point i, due to the load q on the area above point i, is equal to the additional stresses under the point j, due to the load q above the point i. It is also a fact that the additional stresses under the point j, due to the load activeness in point i, is equal to the additional stresses under the point i, due to the load activeness in point i. It can be said that the assumption of additional stress reciprocity is valid.

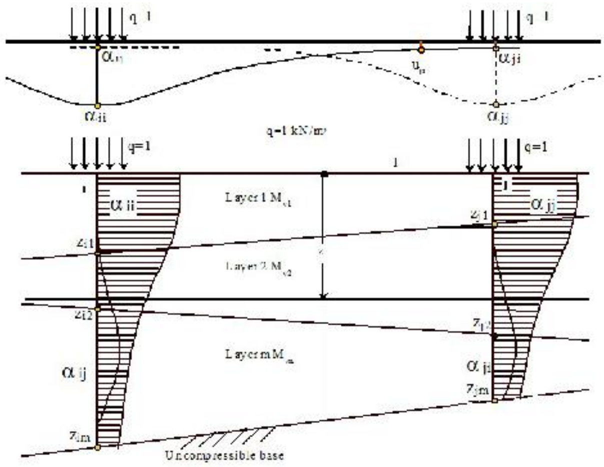


Figure 4. The vertical stresses under the centre element point

In order to determine the deformations and movements underneath the observed points, the real soil stratification, modelled with a suitable modulus of incompressibility of a single layer (linear oedometer soil model), is accepted.

If the additional stresses till any horizontal plane on the depth z are observed, the movements under points i and j due to the activeness of equal load q above the observed points will not be mutually equal $(\alpha_{ii} \neq \alpha_{jj})$ which applies for the movements in point j due to the load in point i and in point i due to the load in point j $(\alpha_{ji} \neq \alpha_{ij})$. Due to the uninformed soil stratification, the theorem of mutual movements is not valid. The same is valid only for homogenous isotropic linear elastic half space. A stratificated half space in general is not homogenous. The influence settlement matrix is unsymmetrical and it is necessary to separately compute its members for every discrete element loaded with single contact pressure.

Model research

Experimental research is composed of observing the loaded girders on different soil thickness. Girders with determined stiffness are placed on engaged layers, which are loaded with different amounts of force. The equipment used in the test is:

a hydraulic press for load application, a micrometer for measuring the vertical deformations, i.e. soil settlements under the girder, an electrically resistant tensometre for measuring the horizontal deformations of the girder due to its bending (HBM 1999). A steel rectangular section has been chosen for the girder and the base has been made from an artificial coherent soil (sand, bentonite, cement and water), which has been placed in a plastic state. The mixture has enabled a high homogeneous degree , with an accomplished extra final hardness after 20 days (cement bonding). The model research has been carried out by using an arranged test model pool with various thickness of "artificial soil" which made a base for beam girders.

The test model construction

The basic test model pool, as a construction for engaging the artificial soil with various thickness, is built in three levels. The reinforced concrete construction is shown on figures 5 and 6.

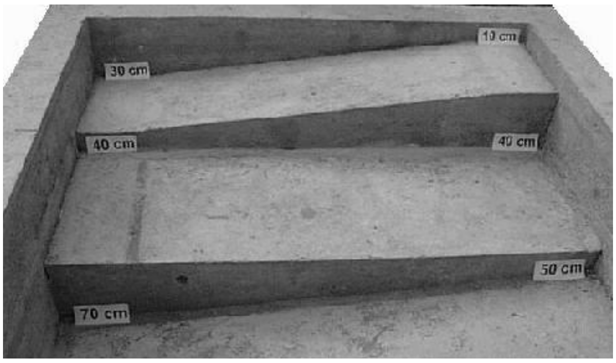


Figure 5. The basic test model pool

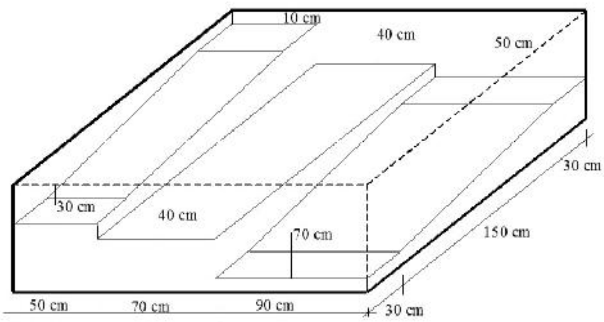


Figure 6. The basic test model pool dimensions

Artificial soil

A mixture made out of prepared patterns with various proportions of the mentioned components is: 87 % of sand; 10 % of bentonite; 3 % of cement and water in an amount of 400 % of bentonite mass. The mixture of satisfying strength, at which a soil failure for the assumed values of load on girder does not take place. It obtained the requested characteristics 20 days after mixing and placing it, in liquid form, in a test mould.

The pool is completely filled with the mixture, which is smoothly graded and covered with wet linen and PVC foil to preserve moisture. Figure 7 shows a test pool filled with artificial soil and made out of a steel construction (HBM 1999).

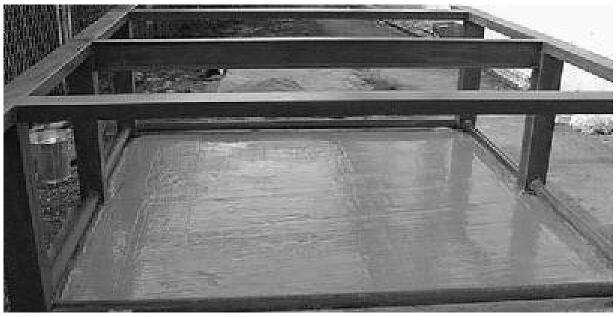


Figure 7. A test pool fill with artificial soil and a steel construction for load

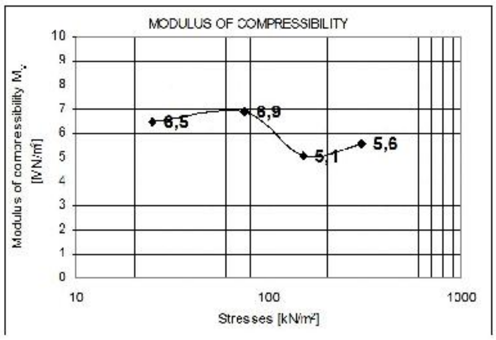
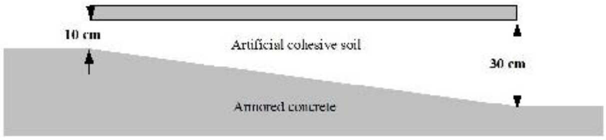


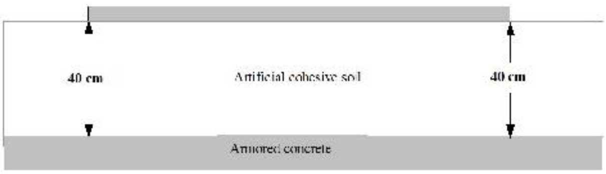
Figure 8. The compressibility diagrams (the stress-modulus of compressibility relation)

The characteristics of artificial soil are also tested by taking the samples from the pool, directly before the research took place, which means 20 days after putting the soil in the pool. The amounts of the modulus of compressibility are shown on the diagram in figure 8. The characteristics are determined by the oedometer test and from the measures made with a test plate of 15 and 30 cm in diameter. Figure 8 shows that the modulus of compressibility values are not uniform, neither in the observed point, nor through the depth of the deformable layer. The soil profiles in model testing There were 3 load girder tests with different soil profiles that took place on prepared bases (figure 9).

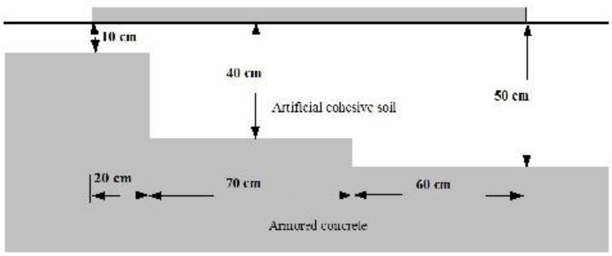
The model testing profiles



Profile I.



Profile II.



Profile III.

Figure 9. The model testing profiles

Girder load

The concrete blocks are used as counter support for presses.

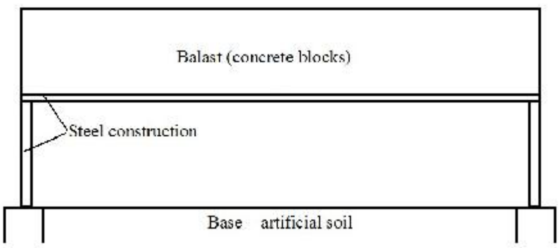


Figure 10. Balast for counter support

The steel girder for model testing

After checking the conditions for satisfying the values of the bend angle for the anticipated amount of forces, a girder with a length L=1,5 m and a cross section b=0,085 m and h=0,050 m is used.

The assumed value of the elastic module of steel was 185 000 MN/ m2. Micrometer. The micrometers are set in 5 points along the girder during the measuring (figure 11). T

he electrically resistant tensometer. During this observation of the steel girder, the electrically resistant tensometers (manufacturer mark: 1-LY13-10/120) measure a base of 10 mm and the resistance of 120 Ω (HBM, 1999).

During the testing, the electrically resistant tensometers are pasted in three profiles, every profile with 4 tensometers (due to the accuracy in deformation measuring in that point) along the girder (L/4, L/2 an 3/4L) which can be seen on figure 11.

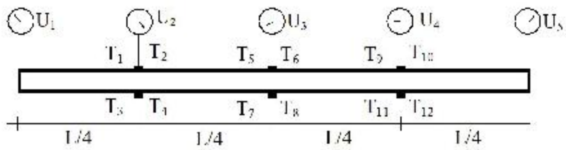


Figure 11. The scheme of placing the micrometer and tensometer along the girder

The forces are applicated on the edges of the girder (figures 12. and 13.).

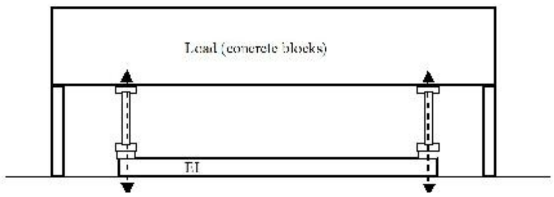


Figure 12. Girder loaded by presses supported on the load

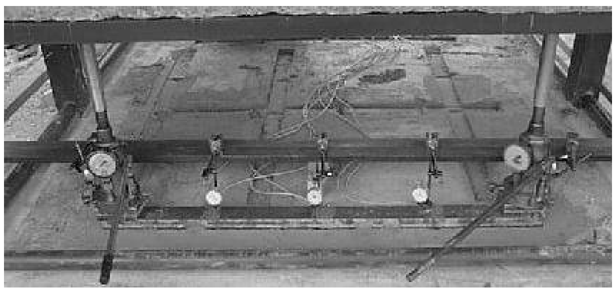


Figure 13. Girder loaded by presses supported on the load

The hydraulic presses had a measure value up to 60 kN and accuracy of 5 kN. The forces were applied in increments of 5 kN up to the force of 40 kN.

The analysis of experimental and theoretical research results

The results of the experimental research and theoretical analysis, gained using the proposed program and their mutual comparison, will be shown.

The profile I is characteristic to the linear change of the soil layer thickness.

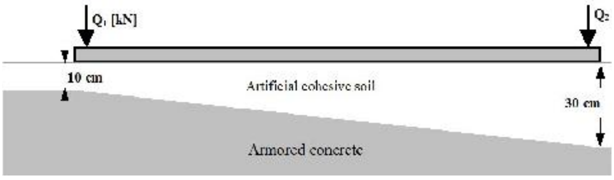


Figure 14. The girder on the deformable soil with linear variation

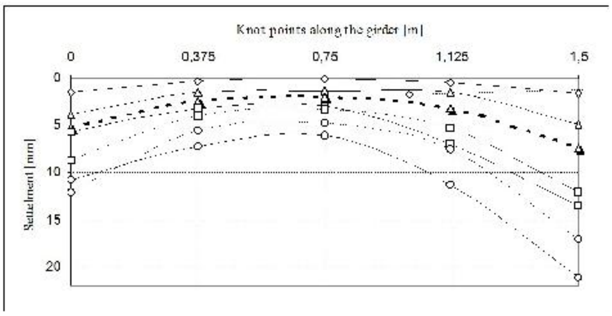


Figure 15. The relation of settlement along the girder and acting forces on girder

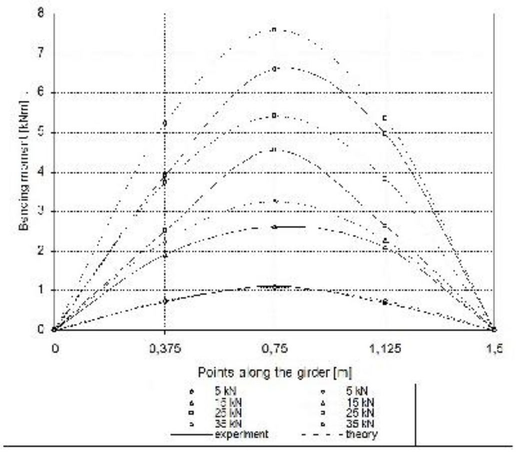


Figure 16. The relation of bending moments along the girder and acting forces on girder

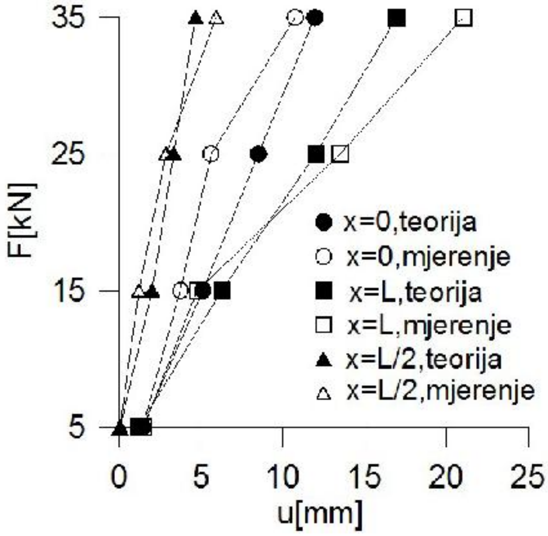


Figure 17. The force-movement diagram o the edges and the middle of the girder.

The calculated movements on the left side, where the soil thickness is relatively small (approximately as the width of the girder), receive greater amounts than the observed, due to the fact that it's not possible to activate the total failure of the base.

The layer becomes stiffer by a gradual increase of load, and a little deformation, compared to the calculated one, computed only using the start modulus of compressibility, takes place.

The situation on the other end (the thickness of the deformable layer is greater) is inverse; the observed movements receive greater amounts. This kind of behaviour is a consequence of soil relief under the girder and the decrease of the competent amounts of modulus of compressibility.

Figure 16 shows the amounts of bending moments along the girder, where it can be seen that the increase of retreat takes place while gradually increasing the outer force.

The profile II with uniform thickness of artificial soil.

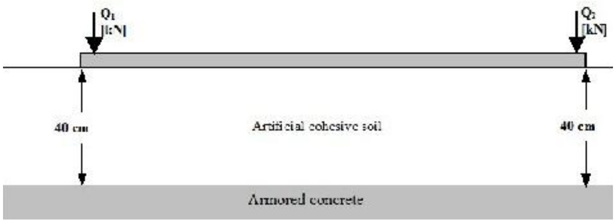


Figure 18. The girder on soil with the same thickness 40 cm

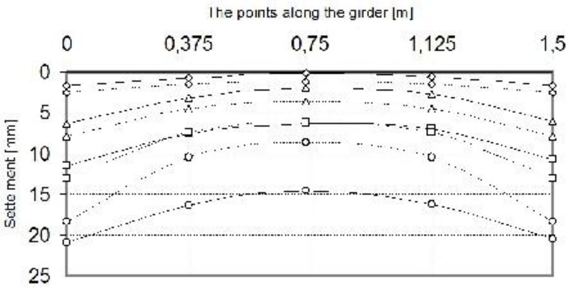


Figure 19. The dependency of settlement along the girder and the acting forces along the girder

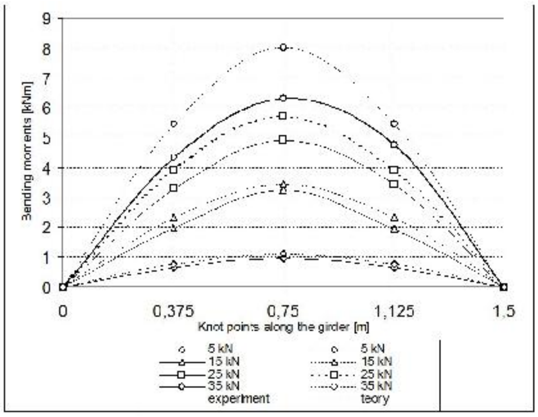


Figure 20. The dependency of the bending moment along the girder and the acting forces on the girder

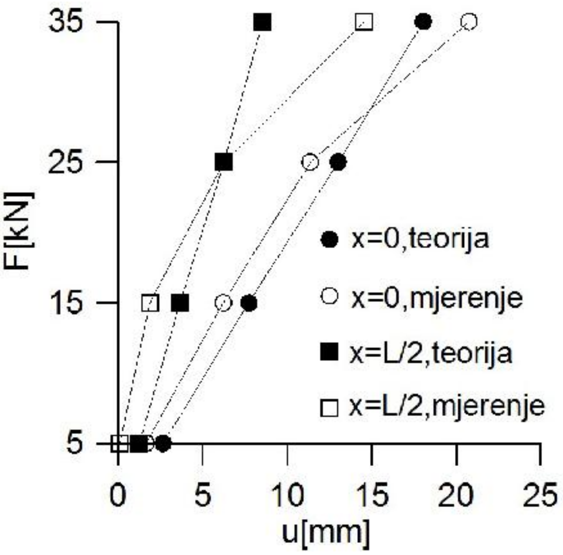


Figure 21. The force-movement diagrams on the edges and in the middle of the girder

The behaviour of the girder is consistent because of the increase in the mutual departure of observed amounts due to the base relief. The calculated bending moments are generally greater than measured, in consequence to the greater relative difference of the movement in the middle and the end of the girder because of the relief made on the points under the total girder length.

The profile III with a leap variation of compressible base thickness

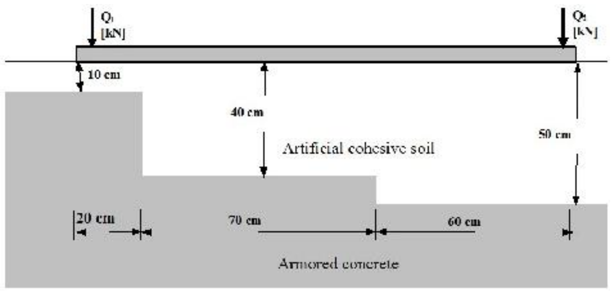


Figure 22. The girder on soil with 10 cm, 40 cm and 50 cm thicknesses

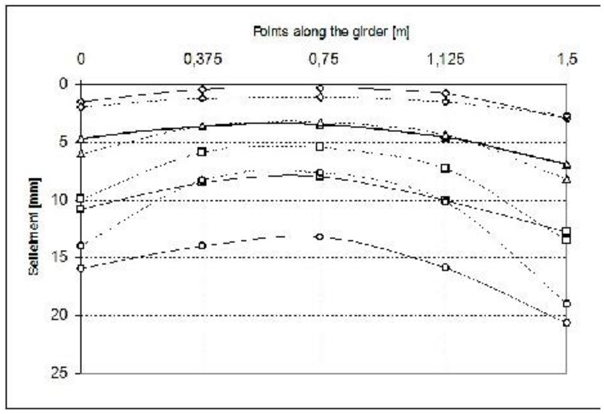


Figure 23. The dependence of settlement along the girder and the forces of acting on the girder
(Forces on the end of girder 5 kN - 15 kN - 25 kN - 35 kN)

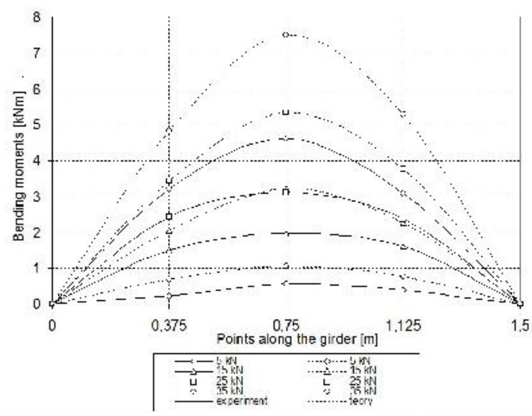


Figure 24. The dependence of settlement along the girder and the forces of acting on the girder
(Forces on the end of girder 5 kN - 15 kN - 25 kN - 35 kN)

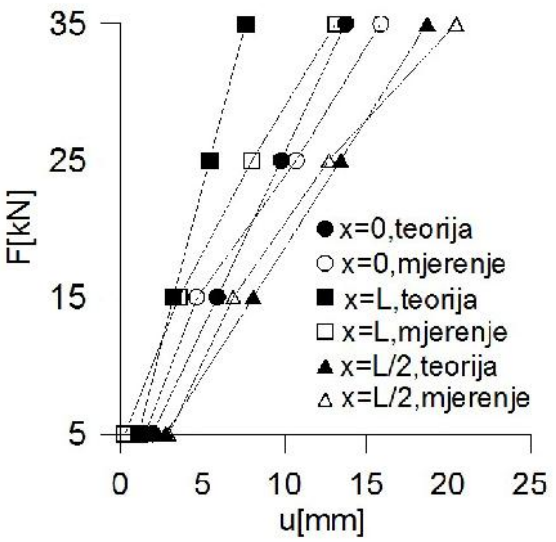


Figure 25. The force-movement diagram on the edges and middle of girder

In the case presented, and on the part of a thick deformable base layer, a creation of plastic deformation takes place, and the measured movements are greater than the ones calculated for greater amounts of force. The mentioned layer is on a relatively short part of the girder (20/150=0.13L), where a sudden leap of the deformable base thickness takes place.

Conclusion

The proposed numerical procedure enables the beam girder design of realistic width on irregular stratified soil. The model test results indicate good conformity with results of the calculated analysis for the amounts of working forces (the break force divided with a corresponding safety factor), which are in the limits of linear elastic soil behaviour.

The perceived departure of the observed occurrences and calculated occurrences take place in the plastic, and particularly in the plastic area of soil deformation. In the elastic deformation area, on the part of the girder, placed on a thick deformable base layer, the calculated movements are greater than the observed ones.

These results are a consequence of the fact that in the thick layer under the girder no soil failure takes place, due to the fact that it's not possible to form a break plane. The layer becomes stiffer and more compact by a progressive increase of outer load amounts, with the amounts modulus of compression greater than at the initial amount.

On the other hand, on the part of the girder that is placed above a deeper deformable layer, movements greater than calculated are observed at the greater amounts of the outer load.

That is a consequence of deformable layer parameters decrease due to the progressive soil relief and the creation of a non-recoverable deformation.

The created program may be useful, explicitly in long beam girder design, where the possibility of occurrence of irregular soil stratification is very great. It is possible to obtain more reliable and economic solutions in comparison with the custom design procedures by considering the true deformable layer geometry.

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Analiza temeljnih nosača na tlu opće uslojenosti

SAŽETAK
Rad prikazuje rezultate istraživanja problema kontakta u sistemu transverzalno opterećenog fleksibilnog grednog nosača i stišljivog, opće uslojenog tla. Rezultati postignute teoretske analize i rezultati modela istraživanja se podudaraju. Razvijeni program može biti iznimno koristan u dizajniranju grednih nosača koji se obično koriste u slučaju nepostojanog sastava tla duž grednog nosača.

Ključne riječi: temeljni nosač, opće uslojeno tlo, eksperiment, slijeganje, moment savijanja, komparacija.